Central University of Haryana

Programme: M.Sc.(Mathematics) — Session:2017-18
Semester: II — Max. Time: 3 hours
Course Title: LINEAR ALGEBRA — Max. Marks: 70
Course Code: SPMMAT 01 02 01 C 3104

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1(a). Let $G : R^3 \to R^3$ be the linear mapping defined by $G(x,y,z) = (x + 2y - z, y + z, x + y - 2z)$. Then find the kernel of G.

(b). Suppose the x and y axes in the plane R^2 are routed counterclockwise 45° so that the new x' and y' are along the line $y = x$ and the line $y = -x$, respectively. Then

(i) Find the change-of-basis matrix P.

(ii) Find the coordinates of the point $A(5,6)$ under the given rotation.

(c). Let A be 4x4 matrix with minimal Polynominal $m(t)=(t^2+1)(t^2-3)$. Find the rational canonical form for A if A is matrix over (i) Q (ii) R, (iii) C.

(d). Prove the following for a linear operator T:

(i) The scalar 0 is an eigen value of T if and only if T is singular.

(ii) If λ is an eigen value of T, where T is invertible, then λ^{-1} is an eigen value of T^{-1}.

(e). Determine all possible Jordan canonical forms J for a linear operator $T : V \to V$ whose characteristic polynomial is $\Delta(t) = (t - 2)^5$ and whose minimal polynomial is $m(t) = (t - 2)^2$.

(f). Let W be the subspace of R^5 spanned by $u = (1,2,3,-1,2)$ and $v = (2,4,7,2,-1)$. Find a basis of the orthogonal complement $W^\perp$ of W.

(g). Consider the polynomials $f(t) = t + 2$ and $g(t) = 3t - 2$ in **P(t)** with inner product $\langle f,g\rangle = \int_0^1 f(t)g(t)dt$. Then find (i) $||f||$ and $||g||$ and (ii) Normalize f and g.

2(a). Let V be a vector space of dimension n. Then prove that (i) any n+1 or more vectors in V are linearly dependent. (ii) Any linearly independent set $S = \{v_1, v_2, \ldots, v_n\}$ with n elements is a basis of V.

(b). Suppose $V = U \oplus W$. Also, suppose $S = \{u_1,...,u_m\}$ and $S' = \{w_1,...,w_n\}$ are linearly independent subsets of U and W, respectively. Then prove that (i) $S \cup S'$ is linearly independent in V. (ii) $Dim.V$ =1, $Dim.U + Dim.W$.

(c). Suppose V has finite dimension and $F : V \rightarrow U$ is linear. Then prove that $Dim.V = Dim.(KerF) + Dim.(Im(F))$.

3(a) Let G be the linear operator on R^3 defined by $G(x,y,z) = (2y + z, x - 2y, 3x)$. Then

(i) Find the matrix representation of G relative to the basis $S = \{w_1,w_2,w_3\} = \{(1,1,1),(1,1,0),(1,0,0)\}$. (ii) Verify that $[G][v] = [G(v)]$ for any vector v in R^3.

(b). Let $T : R^3 \rightarrow R^3$ be defined by $T(x,y,z) = (2x+y-2z, 2x+3y-4z, x+ y - z)$. Then find all eigen values of T, and find a basis of each eigenspace. Is T diagonizable? If so, find the basis S of R^3 that diagonalizes T.

(c). Let V and W be finite dimensional vector space, and p(t) be the minimal polynomial of T. Show that λ is an eigin value of T iff $p(\lambda)=0$

4(a). Let V be a finite dimensional vector space over a field F not of characteristic two. Prove that every symmetric bilinear form on V is diagonalizable.

4(b). Let T: V→V be a linear operator whose characteristic polynomials factors into linear polynomials. Show that V has a basis in which T is represented by a triangle form.

4 (c). State and prove primary Decomposition Theorm.

5(a). Let T be a linear operator on V. Show that each of the following conditions implies $T = \mathbf{0}$:

(i) $\langle T(u),v\rangle = 0$ for every $u,v \in V$.

(ii) V is a complex space, and $\langle T(u),u\rangle = 0$ for every $u \in V$.

(iii) T is self adjoint and $\langle T(u),u\rangle = 0$ for every $u \in V$.

(b). In finite dimensional vector space thereexits unique Adjoint operator for every linear operator T such that $\langle T(x), y\rangle = \langle x, T^*(y)\rangle$.

(c). Prove that the change-of-basis matrix form an orthonormal basis $\{u_1,...,u_n\}$ into another orthonormal basis is unitary. Conversely, if $P = [a_{ij}]$ is a unitary matrix, then prove that the vectors $u'_i = \textstyle\sum_j a_{ji}u_j$ form an orthonormal basis.

CENTRAL UNIVERSITY OF HARYANA
Jant-Pali, Mahendergarh, Haryana
Term End Examination May/June-2018

Programme: M.Sc. Mathematics — Session 2017-18
Semester: II — Max. Time: 3 Hours
Course Title: Mathematical Statistics — Max. Marks: 70
Course Code : SPMMAT 01 02 04 C 3104

Instructions:
(i) Question number 1 has seven sub parts and student need to answer any four. Each sub part carries 3.5 marks.
(ii) Question number 2 to 5 have three sub parts and student need to answer any two. Each sub part carries 7 marks.

1. Attempt any four.
   (a) Define skewness and kurtosis. [3.5]
   (b) For any two events A and B, show that $P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$. [3.5]
   (c) Find $E(X)$ if X be a continuous random variable with p.d.f. $f_X(x) = 5x^4$ for $0 < x < 1$ and $f(x) = 0$ otherwise. [3.5]
   (d) Eight coins are tossed simultaneously. Find the probability of getting atleast six heads. [3.5]
   (e) State Central limit theorem and what are its applications ? . [3.5]
   (f) Define exponential distribution and obtain its M.G.F. [3.5]
   (g) Define Null and Alternate hypothesis with suitable example. [3.5]

2. (a) Determine which of the following are probability functions ?

$$f(x) = \begin{cases} 3x, & 0 < x \leq 1; \\ 9 - 3x, & 1 < x < 3. \\ 0, & \text{otherwise.} \end{cases} \quad g(x) = \begin{cases} 4/5, & x = 1; \\ 1/5, & x = 2; \\ 0, & \text{otherwise.} \end{cases}$$

[7]

   (b) Find the mode of the following frequency distribution

X:	1	2	3	4	5	6	7	8	9	10	11	12
f:	3	8	15	23	35	40	32	28	20	45	14	6

[7]

   (c) Ten students got following percentage of marks in Algebra and Topology. Find coefficient of correlation [7]

Student	Marks in Algebra	Marks in Topology
1	78	84
2	36	51
3	98	91
4	25	60
5	75	68
6	82	62
7	90	86
8	62	58
9	65	53
10	39	47

3. (a) (i) Does there exists any random variable whose expected value is not finite ? Explain with suitable example.
(ii) Give an example to show that for a random variable X if there exist a finite mean, $E[x^2]$ may not be finite. [3.5+3.5]

(b) (i) Probability of hitting a target in any attempt is 0.6, what is the probability that it would be hit in fifth attempt.
(ii) Determine the geometric distribution for which mean is 3 and variance is 4. [3.5+3.5]

(c) Define Poisson's distribution. Obtain its cumulant generating function and show that all cumulants are equals.

4. (a) If a r.v. X follows uniform distribution, find mean and variance . [7]

(b) Show that Normal distribution is a limiting case of Binomial distribution. [7]

(c) Find the expression for even orders moments about mean and show that odd order moments vanish. [7]

5. (a) Define the following terms (i) Test statistics (ii)Significance level (iii)Critical value (iv)Type I and Type II errors . [7]

(b) An urn contains either 4 white and 2 black balls or 2 white and 4 black balls. Two balls are to be drawn from the urn. If less than two white balls are obtained, it will be decided that this urn contains 2 white and 4 black balls. Calculate the values of α and β. [7]

(c) Suppose $X_1, X_2, ..., X_n$ is a random sample taken from normal population with mean μ and variance σ^2. The following two estimators are suggested to estimate μ as $T_1 = \frac{X_1+X_2+X_3}{3}$ and $T_2 = \frac{X_1+X_2}{4} + \frac{X_3}{2}$. Are both estimators unbiased ? Which one of them is more efficient ? [7]

CENTRAL UNIVERSITY OF HARYANA
Jant-Pali, Mahendergarh, Haryana
Term End Examination May-2018

Programme: MSc Mathematics
Semester: II
Course Title: Mathematical Statistics
Course Code : SPMMAT 01 02 CC 04 3104

2017-18 Rejoin
Max. Time: 3 Hours
Max. Marks: 60

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1. (i) What are various measures of central tendency. Explain in detail.
   (ii) Discuss different ways of graphical representation of frequency distribution. [6+6]

2. (i) If a r.v. X follows uniform distribution, find mean and variance .
   (ii) Define Binomial distribution and its applications in details. [6+6]

3. (i) If χ_1^2 and χ_2^2 are two independent χ^2 variates with n_1 and n_2 degrees of freedom, then show that $\frac{\chi_1^2}{\chi_2^2}$ is a $\beta_2(\frac{n_1}{2}, \frac{n_2}{2})$ variate.
   (ii) Find moment generating function for χ^2 distribution. [6+6]

4. (i) Probability of hitting a target in any attempt is 0.6, what is the probability that it would be hit in fifth attempt.
   (ii) Determine the geometric distribution for which mean is 3 and variance is 4. [6+6]

5. (i) If a r.v. X follows Gamma distribution, find mean and variance .
   (ii) Show that poisson distribution is a limiting case of Binomial distribution. [6+6]

6. (i) State and prove Chebychev inequality.
   (ii) Find mean deviation from arithmetic mean for the following data

X	30	31.5	33	34.5	36	37.5	39	40.5
f	4	19	30	63	66	29	18	1

[6+6]

7. (i) If two dice are thrown, what is the probability that the sum is
   (a) greater that 8, (b) neither 7 nor 11 ?
   (ii) A candidate applies for a job in two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected at Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the firms ? [6+6]

8. Show that Normal distribution is a limiting case of Binomial distribution. [12]

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations May/June 2018

Programme: - M.Sc. Mathematics **Session: 2017-18**

Semester: II **Max. Time: 3 Hours**

Course Title: Topology **Max. Marks: 70**

Course Code: SPMMAT 01 02 02 C 3104/SPM MAT 01 02 CC 02 3104

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each subpart carries three and half marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two subparts of each question. Each sub part carries seven marks.

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1. Answer any four from the following seven sub parts: $(4 \times 3.5 = 14)$

   (a) Give an example of an ordered set X with order topology in which (a,b) is proper subset of $[a,b]$, where (a,b) contains infinitely many points of X. Justify your answer.

   (b) Prove that if every finite set is closed in a topological space X then for each pair of points of X, each has a neighborhood not containing the other.

   (c) Establish that $[0,1)$ as a subspace of R is not homeomorphic to the unit circle S^1, considered as a subspace of R^2.

   (d) If $f: X \to Y$ is continuous, where X is compact and Y is Hausdorff, then show that f is a closed map.

   (e) Show that if A is a connected subspace of X, then $\bar{A}$ is also connected subspace of X. Is the converse true? Justify.

   (f) Show that if X is a space having a countable basis β, then any discrete subspace A of X must be countable.

   (g) Give an example of a space that is Hausdorff but not regular. Justify.

2. Answer any two sub parts from the following: $(2 \times 7 = 14)$

   (a) In an infinite set X, if every infinite subset of X is open with respect to T, then T is the discrete topology on X.

(b) Let $\beta = \{[a,b) \mid a < b,\ a$ and b rational$\}$ be a basis for a topology on R. Is the topology generated by β different from R_l? Justify your answer. Also find

$\overline{(0,\sqrt{3})}$ in the topology generated by β.

(c) Show that X is Hausdorff if and only if the subset $\{(x,x) \mid x \in X\}$ of $X \times X$ is closed in $X \times X$.

[P.T.O]

3. Answer any two sub parts from the following: $(2 \times 7 = 14)$

(a) Let X be a topological space and $A \subset X$. If there is a sequence of points of A converging to x, then prove that $x \in \overline{A}$. Show that the converse need not be true.

(b) Let Y be an ordered set in the ordered topology. If $f, g : X \to Y$ are two continuous maps, then show that the map $h : X \to Y$ defined as

$$h(x) = min\{f(x), g(x) \mid x \in X\}$$

is continuous.

(c) Let $f_n : X \to Y$ be a sequence of continous functions from the topological space X to the metric space Y. Show that if $<f_n>$ converges uniformly to f, then f is continuous.

4. Answer any two sub parts from the following: $(2 \times 7 = 14)$

(a) A topological space is totally disconnected if the connected components are allsingletons. Prove that any countable metric space is totally disconnected.

(b) Give example of a topological space which is connected but not locally connected and vice versa.

(c) Show that [0,1] is not limit point compact as a subspace of R_l, the lower limit topology.

5. Answer any two sub parts from the following: $(2 \times 7 = 14)$

(a) Show that if X is Lindel¨of and Y is compact, then $X \times Y$ is Lindel¨of. If Y is Lindel¨of but not compact, then can we say $X \times Y$ is Lindel¨of? Justify your answer.

(b) If X is a T_1-space, then prove that every countable open covering of X contains a finite subcollection that covers X if and only if every infinite subset of X has a limit point in X.

(c) State and prove Urysohn Lemma.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2018

Programme	: M.Sc. Mathematics		
Semester	: II	Session	: 2017 – 18
Course Title	: Numerical Analysis	Max. Time	: 3 Hours
Course Code	: SPMMAT 01 02 03 C 3104	Maximum Marks	: 70

Instructions:
(i) Question number 1 has seven sub parts and student need to answer any four. Each sub part carries 3.5 marks.
(ii) Question number 2 to 5 have three sub parts and student need to answer any two. Each sub part carries 7 marks.

1. (a) Obtain the approximate solution of IVP $y' = 1+ty$, $y(0) = 1$ using Picard's method. (4X3.5=14)
   (b) Define absolute and relative errors.
   (c) Write Stirling and Bessel formulas.
   (d) Find $\int_0^1 (99x^2 - x)\, dx$ using Simpson 1/3 rule. Does change in size of h improve the solution? Justify? Find error also.
   (e) Discuss any direct method for solution of system of linear equations.
   (f) Classify $2nd$ order PDEs. What is D'Alembert solution to the wave equation?
   (g) What is the degree of a polynomial from which the following data is taken:

x	-2	-1	0	1	2	3
p(x)	-5	1	1	1	7	25

2. (a) Find root of $x^3+x-1=0$ using Newton-Raphson method. Under what conditions N-R method converge?
   (b) Consider the system of equations

$$\begin{bmatrix} 1 & -2 & 5 \\ 5 & 2 & -1 \\ 2 & 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 6 \\ 5 \end{bmatrix}$$

   What should be initial values of (x^0, y^0, z^0) so that Gauss-Jacobi method converges? Find second iterated values (x^2, y^2, z^2), if $(x^0, y^0, z^0) = (1, 1, 0)$.
   (c) The distance traversed by a train(y), at different time t, are obtained as:

t	1	3	5	8	10
y	10	40	85	125	200

   Assuming the relation $y = at^2+b$, find the best values of a and b. In what term you are claiming the values of a and b are best?.

3. (a) Show that there exists a unique polynomial of degree atmost n interpolating $n+1$ data points.
   (b) Find the difference between the values obtained by Newton forward and Gauss forward formulas for $y(3.5)$ for the given data:

x	0	1	2	3	4	5	6
y	-4	-4	4	54	212	556	1196

(c) Find the minimum value of y in $[0.60, 0.75]$ interval from the given data:

x	0.60	0.65	0.70	0.75
y	0.6221	0.6155	0.6138	0.6170

What is the value of x where y is minimum?

4. (a) Use one and two-point Gauss Quadrature Rule to approximate the distance covered by a rocket from $x = 8$ to $x = 30$ as given by $d = \int_8^{30}(20x^2 + 9.8x)\,dx$.

(b) Solve $\frac{dy}{dx} = 1 + y^2$ at $x = 0.8$ by Adam-Bashforth Moulton's predictor-corrector method, where $y(0) = 0, y(0.2) = 0.2048, y(0.4) = 0.4269$. Hint: First find $y(0.6)$ using R-K fourth order method.

(c) A river is 80m wide. The depth y for the river at a distance x from one bank is given in following table:

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	9

Find approximate area of cross section of river using Romberg's integration formula.

5. (a) Derive five point formula for solution of Laplace equation.

(b) Using Crank-Nicolson formula, solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ in a rod of length 8 units under following conditions $u(0;t) = 0, u(8,t) = 100, u_{1,0} = u(2,0) = 50, u_{2,0} = u(4,0) = 150, 4_{3,0} = u(6,0) = 200$. Find the values at $u_{1,1}, u_{2,1}$ and $u_{3,1}$. Hint: $h = 2$.

(c) Solve the differential equations $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$ at $x = 0(1)5, t = 0(1)5$, when $u(0,t) = 0$, $u(5,t) = 10\,t, u(x,0) = 0, \dfrac{\partial u}{\partial t}(x,0) = \dfrac{x(5-x)}{10}$.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2018

Programme	: M.Sc. Mathematics (Re-Join) (Re-Appear)		
Semester	: Second	Session	: 2015 – 16
Course Title	: Measure Theory and Integration	Max. Time	: 3 Hours
Course Code	: SPMMAT 01 02 DCEC 02 3104	Maximum Marks	: 60

Note: Attempt any five questions.

1. (a) Prove that the outer measure of an interval is its length. [6]
   (b) Give example of non measurable subset of $\mathbb{R}$. [6]

2. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers, then prove countable additive property $m^*(\bigcup A_n) \leq \sum m^*(A_n)$. [6]
   (b) Prove that every countable set has measure zero. Does converse holds. [6]

3. (a) If $E_i \in \mathscr{B}$ and $E_i \subset E_{i+1}$, then

$$\mu\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu(E_n).$$

[6]

   (b) Define measurable function. Show that both step and simple functions are measurable. [6]

4. (a) Let f be a measurable function. Then show that both f^+ and f^- are measurable. [6]
   (b) Show that the characteristic function χ_A is measurable if and only if A is measurable. [6]

5. (a) When a function is Riemann integrable and when it is Lebesgue integrable? What is major difference in both definitions? [6]
   (b) Let ϕ and ψ be simple functions which vanish outside of a set of finite measure, then show that [6]

$$\int (a\phi + b\psi) = a\int \phi + b\int \psi$$

and if $\phi \geq \psi$ a.e., then

$$\int \phi \geq \int \psi.$$

6. (a) Show that the space $C[a, b]$ is a dense subset of $L_1[a, b]$. [6]
   (b) Let f and g are bounded measurable functions defined on a set E of finite measure, then [6]
      i. if $f = g$ a.e. then $\int_E f = \int_E g$
      ii. if $A \leq f(x) \leq B$ then $A \cdot m(E) \leq \int_E f \leq B \cdot m(E)$

7. (a) State Monotone convergence theorem and Fatou's lemma for integration.. [6]
   (b) State and prove Lebesgue Dominated convergence theorem. [6]

8. (a) Define convergence, pointwise convergence, almost everywhere convergence and convergence in measure of a sequence of measurable functions. Show that a.e. limit of sequence of measurable functions on a measurable set E, is also measurable on E. [6]
   (b) State Little Wood's three principles. [6]

CENTRAL UNIVERSITY OF HARYANA
Jant-Pali, Mahendergarh, Haryana
Term End Examination May-2018

Programme: M.Sc. Mathematics
Semester: IV
Course Title: Finite Element Analysis
Course Code : SPMMAT 01 04 DCEC 03 3104

2017-18 Rejoin
Max. Time: 3 Hours
Max. Marks: 60

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1. Obtain approximate solution of $v'' - \frac{k}{T}v + \frac{f}{T} = 0$, $v(0) = v(L) = 0$ with $T = 600\ lb, L = 120\ in, k = 0.5\ lb/in^2, f = 2\ lb/in$ using trial function $a_0(x^2 - xL) + a_1(x^4 - xL^3)$. [12]

2. Using Simple natural coordinates, find Shape functions for nine node rectangular element. [12]

3. Using Galerkin method, formulate and solve the FEM model for standard one dimensional transient heat conduction problem. [12]

4. Derive Shape functions for Hexahedron element. [12]

5. Find approximate solution for $TV'' - K.V(x) = -f(x)$, $V(0) = 0$, $V(L) = 0$ using the trial solution $v = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$ by using Galerkin approach for $T = 600, K = 1/2, L = 120, f = 2$. [12]

6. Derive three point Gauss Quadrature rule. [12]

7. Compute two parameter Ritz solution for

$$-\frac{d}{dx}[(1+x)\frac{du}{dx}] = 0 \quad u(0) = 0, u(1) = 1, \text{ for } 0 < x < 1$$

using $\phi_0 = \sin \pi x/2$ and $\phi_i = \sin i\pi x$. [12]

8. Explain various weighted residual approaches. [12]

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations May /June 2018

Programme: M.Sc. Mathematics **Session: 2017-18**

Semester: IV **Max. Time: 3 Hours**

Course Title: Mathematical Modelling **Max. Marks: 70**

Course Code: SPMMAT 01 04 02 DCEC 3104

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.

2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. (4X3.5=14)

a) Draw the trajectories of the model

$$\frac{dx}{dt} = x(1-2y)$$
$$\frac{dy}{dt} = -y(1-3x)$$

b) If Newton's law of cooling is assumed to be true for a body of temperature initially at 300 degrees Celsius when placed in a large block of ice, then determine its temperature at end of 5 min.

c) Discuss the behavior of the solution of the equation $x_{t+2} - 7x_{t+1} + 12x_t = 0$ as $t \to \infty$.

d) Show that the logistic model can be written as $\frac{1}{N}\frac{dN}{dt} = r\frac{(K-N)}{K}$ and prove that K is the limiting value of the population.

e) Discuss Prey-Predator model.

f) Write a short note on directed signed graph.

g) Solve the difference equation $y_{k+1}=2y_k + 3$; k=0,1,2,....

Question No. 2. (2X7=14)

a) Consider a generalized model of a population growth in the form

$$\frac{1}{N}\frac{dN}{dt} = \frac{r}{\alpha}\left(1-\left(\frac{N}{K}\right)^{\alpha}\right), \alpha > 0$$

Then, solve the above equation for N and find the limiting value of population level.

b) Show that the function $u(x,t) = f(x+ct) + g(x-ct)$ satisfy the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}$$

c) If a fish population is assumed to grow logistically and at the same time is assumed to be harvested at a constant rate H, then determine all possible value of limiting population of fish.

Question No. 3. (2X7=14)

a) Solve the following simultaneous difference equation

$$x_{n+1} - 2\,x_n - y_n = n$$

$$y_{n+1} - 2\,x_n - 3y_n = -n$$

b) Consider a competition model for the species whose population at any time t is governed by equations

$$\frac{dx}{dt} = x(A_1 - B_1 x - C_1 y)$$

$$\frac{dy}{dt} = y(A_2 - B_2 y - C_2 x)$$

Where $A_1, B_1, C_1, A_2, B_2, C_2$ are all positive constants, then,

1. Show that equilibrium point will be biologically feasible if the $\frac{B_2}{C_2} > \frac{A_2}{A_1} > \frac{C_2}{B_1}$.
2. If it is biologically feasible then find the condition so that the equilibrium point will be stable.

c) Find the solution of the autonomous second order difference equation

$y_{t+2} + ay_{t+1}\ by_t$ =c , where t=0,1,2... and a, b, c are some constants.

Question No. 4. (2X7=14)

a) Explain Cobweb model of supply and demand.

b) A cup of coffee is initially at boiling point $100^{o}C$. The temperature of room is $20^{o}C$. Find the temperature of coffee as a function of time.

c) Consider the equation $y_n = a\,y_{n-1} + b\,n + c$, where a, b and c are constants, then

1. Find the solution of the difference equation,
2. Find the asymptotic solution for $n \to \infty$, for the cases $a < 0$, $a = 0$ and $a > 0$.

Question No. 5. (2X7=14)

a) Explain Konisberg bridge problem and why it is not solvable?

b) Define weighted graph and discuss any algorithm for shortest path problem with suitable example.

c) Discuss travelling salesman problem.

CENTRAL UNIVERSITY OF HARYANA

Term End Examinations May /June 2018

Programme: **M.Sc. Mathematics** **Session: 2016-17**

Semester: **IV** **Max. Time: 3 Hours**

Course Title: **Advanced Complex Analysis** **Max. Marks: 70**

Course Code: **SPM MAT 01 04 05 DCEC 3104**

Instructions:

1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

Question No. 1. (4X3.5=14)

a) Prove that every function which is meromorphic in the whole complex plane, is the quotient of two entire functions.

b) Using Hadamard's three circle theorem, prove that M(r) = $\max_{|z|\leq r}|f(z)|$ is a convex function of lnr.

c) State Little Picard's theorem. Give an example of a function which is entire and non-constant such that it does not assume one value in the whole complex plane with proper justification.

d) Show that a differential function f on [a,b] is convex if f' is increasing.

e) Let f(z) be analytic in a region R and suppose that f(z) = 0 at all points on an arc PQ inside R. Prove that f(z) = 0 throughout R.

f) Explain Riemann mapping theorem.

g) Let G be an open set in $\mathbb{C}$ and H(G) be the space of all analytic functions on G. Then if $\{f_n\}$ is a sequence in H(G) and f is a continuous function from G into $\mathbb{C}$ such that $f_n \to f$ then prove that f is analytic in G.

Question No. 2. (2X7=14)

a) State and prove Phragmen-Lindelof theorem.

b) Show that if f: (a,b) → R is convex then f is continuous. Does it remain true if it is defined on [a,b].

c) Prove that $\cot z = \frac{1}{z} + 2z\left\{\frac{1}{z^2-\pi^2} + \frac{1}{z^2-4\pi^2} + \cdots\right\}$.

Question No. 3. (2X7=14)

a) State and prove Weistrass factorization theorem.

b) Let G be an open set in $\mathbb{C}$ and (Ω, d) be a complete metric space then prove that the space of all continuous functions from G into Ω is a complete metric space.

c) Let G be a region and suppose that the sequence $\{f_n\}$ in H(G), the space of all analytic functions in G, converges to f. If f $\neq$ 0, a closed ball $\bar{B}$ (a,r) $\subset G$ and f(z) $\neq 0$ for |z-a|=R(>r) then prove that $\exists$ an integer n_0 such that for n $\geq n_0$, f and f_n have the same number of zeroes in B(a,r).

Question No. 4. (2X7=14)

a) State and prove Mittag Leffler's theorem.

b) Prove that if f(z) is an entire function and f(0) $\neq$ 0, then

$$f(z) = f(0)\, P(z)\, e^{g(z)}$$

where P(z) is a product of primary factors and g(z) is an entire function.

c) Let f be a holomorphic function on the horizontal half strip

$$\left\{z: \frac{-\pi}{2} \leq y \leq \frac{\pi}{2} \text{ and } 0 \leq x\right\}.$$

If $|f(z)| << e^{e^{cRe(z)}}$, for some constant $0 \leq c < 1$ then prove that $|f(z)| \leq 1$ on the edges of the half-strip implies $|f(z)| \leq 1$ in the interior, as well.

Question No. 5. (2X7=14)

a) State and prove maximum modulus theorem for harmonic functions.

b) State and Prove Jensen's formula.

c) Find a function harmonic inside the unit circle |z|=1 and taking the prescribed values given by

$$F(\theta) = \begin{cases} 1, & 0 < \theta < \pi \\ 0, & \pi < \theta < 2\pi \end{cases}.$$

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2018

Programme	: M.Sc. Mathematics (Re-Appear)		
Semester	: Third	Session	: 2016 – 17
Course Title	: Applied Discrete Mathematics	Max. Time	: 3 Hours
Course Code	: SPMMAT 01 03 04 DCEC 3104	Maximum Marks	: 70

Instruction:
1. Question number 1. has seven sub-parts and student need to answer any four. Each subpart carries 3.5 marks.
2. Question number 2 to 5 have three sub parts and students need to answer any two subparts of each question. Each subpart carries seven marks.

1. (a) Define complete bipartite graph $K_{n,m}$. (4X3.5=14)
   (b) What do you mean by Tautology and Fallacy statement?
   (c) What is Konigsberg Bridge problem. Is it solvable? Why?
   (d) Define Lattice as a algebraic system.
   (e) Give example of a lattice which is not a Boolean Algebra. Justify.
   (f) Check whether the set of all divisors of 20 with division relation is a Boolean algebra.
   (g) Define proposition with example.

2. (a) Check the validity of following arguments:
   If I run I will get there quicker
   I did not run
   Therefore I did not get there quicker.

   and

   If I run I will get there quicker
   I got there quicker
   Therefore I must have ran.
   What are Modus Ponens and Modus Tollens rules of inference. Discuss their validity also.
   (b) Define and find converse, inverse, contra-positive and negation of $(p \wedge q) \rightarrow (r \vee s)$. What will be the negation of:
   There is some student in our university, none of whose friends are also friends of each other.
   (c) Show that $\sim (p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent.

3. (a) Show that both the definitions of lattice as a partially ordered set and as a algebraic system are equivalent.
   (b) Define partial ordered set, maximal and greatest element. Find maximal, minimal and greatest elements, if any, of poset $(\{2, 4, 5, 10, 12, 20, 25\}, |)$.
   (c) Define distributive and complemented lattice. Show that in a finite complemented distributive lattice, every element a is the join of a unique set of atoms.

4. (a) Show that every Boolean algebra is Distributive, complemented and bounded lattice.
   (b) Show that every finite boolean algebra has cardinality 2^n for some n.
   (c) Express the logic circuit with minimal OR gates for $wx\overline{yz} + w\overline{x}yz + w\overline{x}y\overline{z} + \overline{w}x\overline{yz} + \overline{wx}y\overline{z} + \overline{wxyz}$.

5. (a) Discuss any one algorithm to find minimal spanning tree.

(b) Define path in a graph. What is difference between Eulerian and Hamiltonian path? Show that Eulerian circuit exist if and only if degree of each vertex is even.

(c) Define Adjacency matrix. Show that the number of paths of length k from vertex i to vertex j is $(i, j)^{th}$ entry of matrix A^k.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2018

Programme	: M.Sc. Mathematics (Re-Appear)		
Semester	: Third	Session	: 2016 – 17
Course Title	: Functional Analysis	Max. Time	: 3 Hours
Course Code	: SPMMAT 01 03 02 C 3104	Maximum Marks	: 70

Instruction:
1. Question number 1. has seven sub-parts and student need to answer any four. Each subpart carries 3.5 marks.
2. Question number 2 to 5 have three sub parts and students need to answer any two subparts of each question. Each subpart carries seven marks.

1. (a) State Closed Graph Theorem. (4X3.5=14)
   (b) Define Self-adjoint, Unitary and Normal operators.
   (c) $C[0,1]$ with sup norm is Banach Space. Show that it is not a Hilbert space.
   (d) Define Hilbert adjoint operator. Show that $(ST)^* = T^*S^*$.
   (e) Define Dual and algebraically dual spaces.
   (f) Show that if A is compact subset of an infinite dimensional normed space X then A is nowhere dense in X.
   (g) Give example of a linear map which is not bounded.

2. (a) Determine which of the following is norm on $\mathbb{R}$:
   (i) $||x|| = 2|x|$ (ii) $||x|| = e^x$
   (b) Show that the space $B(X,Y)$ of all bounded linear maps from X to Y, where Y is complete, is complete.
   (c) In a finite dimensional normed linear space X, a subset A of X is compact if and only if A is closed and bounded. Does this result hold in infinite dimensional normed spaces also? Give example.

3. (a) Define Fourier coefficients. Show that any vector x in an Inner product space can have countably many non-zero Fourier coefficients w.r.t an orthonormal family $< e_n >; n \in I$, in X.
   (b) Define Hilbert dimension. Show that two Hilbert spaces over same field are isomorphic if and only if they have same Hilbert dimension.
   (c) Let X be Hilbert space, K a nonempty convex subset of X and $x \in X$. Then show that $\exists!$ $x_0 \in K$ that minimizes the distance from x to K, i.e., $||x - x_0|| = dist(x, K) \equiv inf_{y \in K}||x - y||$.

4. (a) Show that every projection operator on a Hilbert space H is self-adjoint. Conversely, show that every self-adjoint operator that is idempotent on H is a projection.
   (b) Let $X = (C_{00}, ||\cdot||_2)$ and $f : X \to \mathbb{R}$ be a bounded linear functional $f(x) = \sum_{n=1}^{\infty} \frac{x_n}{n}$. Show that the operator $T : X \to X$ defined as $T(< x >) =< f(x), 0, 0, \ldots >$ is bounded but adjoint operator T^* does not exist.
   (c) Define weak and strong convergence of $< x_n >$ in a NLS X. Show that strong limit, if exist, of a sequence of bounded self-adjoint linear operators is also bounded self-adjoint linear operator.

5. (a) State and prove uniform boundedness principle.

(b) Let X be real vector space and p a sublinear functional on X. Let Z be subspace of X and $f : Z \to \mathbb{R}$ be a linear functional satisfying $f(x) \leq p(x)$, then show that f has an extension $\tilde{f} : X \to \mathbb{R}$ satisfying $\tilde{f}(x) \leq p(x)$.

(c) State and prove Open Mapping Theorem.

CENTRAL UNIVERSITY OF HARYANA
Jant-Pali, Mahendergarh, Haryana
Term End Examinations May/June-2018

Name of Programme	: M.Sc. Mathematics		
Semester	: Second Semester **(Re-Appear)**	Session	: 2015-2017
Course Title	: Linear Algebra	Max. Time	: 3 Hours
Course Code	: SPMMAT 01 02 CC 01 3104	Max. Marks	: 60

Note: Attempt any **five** questions.

1. (a.) State and prove Replacement theorem. [6]

   (b.) Consider the subspaces $W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = a_2 + 2a_3\}$ and $W_2 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1 = 3a_2 + 2a_3\}$ of $\mathbb{R}^3$. What are the dimension of $W_1 + W_2$? [6]

2. (a.) Let V and W be vector spaces, and let $T : V \to W$ be linear. If V is finite dimensional, then $nullity(T) + rank(T) = dim\ (V)$. [6]

   (b.) Let S be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Then $S \cup \{v\}$ is linearly dependent if and only if $v \in span(S)$. [6]

3. Let V and W be finite-dimensional vector spaces with ordered bases β and γ, respectively. Let $T : V \to W$ be linear. Then T is invertible if and only if $[T]_\beta^\gamma$ is invertible. Furthermore, $[T^{-1}]_\gamma^\beta = ([T]_\beta^\gamma)^{-1}$. [12]

4. Let V be a finite dimensional inner product space, and let T be a linear operator on V. Then there exists a unique linear operator T^* on V such that [12]

$$< T(x), y >=< x, T^*(y) > \quad \forall x, y \in V.$$

5. State and prove Gram-Schmidt orthogonalization process. [12]

6. (a.) State and prove Cayley Hamilton Theorem. [6]

   (b.) Suppose A is the change-of-basis matrix from a basis $\{v_i\}$ to a basis $\{w_i\}$, and suppose B is the change-of-basis matrix from the basis $\{w_i\}$ back to $\{v_i\}$. Prove that A is invertible and that $B = A^{-1}$. [6]

7. Let $T : V \to V$ be a linear operator whose characteristic polynomial factors into linear polynomials. Then V has a basis in which T is represented by a triangular matrix. [12]

8. (a.) Let V be a finite dimensional vector space over a field F not of characteristic two. Prove that every symmetric bilinear form on V is diagonalizable. [6]

   (b.) Determine all possible Jordan canonical forms for a linear operator $T : V \to V$ whose characteristic polynomial $p(t) = (t-3)^3(t-6)^2$. In each case, find the minimal polynomial. [6]

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2018

Programme	: M.Sc. Mathematics (Re-Appear)	Session	: 2016 – 17
Semester	: Third	Max. Time	: 3 Hours
Course Title	: Mechanics	Maximum Marks	: 70
Course Code	: SPMMAT 01 03 05 C 3104		

Instruction:
1. Question number 1. has seven sub-parts and student need to answer any four. Each subpart carries 3.5 marks.
2. Question number 2 to 5 have three sub parts and students need to answer any two subparts of each question. Each subpart carries seven marks.

(4X3.5=14)

1. (a) Define Equimomental systems.
   (b) Prove that the transformation $\tilde{q}_i = \alpha q_i$, $\tilde{p}_i = \beta \tilde{p}_i, (i = 1, 2, \ldots, n)$ is canonical.
   (c) What is difference between free and constrained system?
   (d) Define cyclic co-ordinates.
   (e) Explain Lagrange Brackets? How they differ from Poisson Brackets?
   (f) What are Holonomic and non-Holonomic system of particles?
   (g) Define Hamiltonian function.

2. (a) Define Moments and product of Inertia. State and prove theorem of parallel axis.
   (b) State the theorem of perpendicular axes. Find the moment of inertia about any line through the meeting point of the perpendicular axes, where the moments and product of inertia about these three axes being known.
   (c) Define Principal axes and prove that principal axis are mutually orthogonal. Also, find the principal moment of inertia of a rigid body.

3. (a) Drive Lagranges equations for impulsive forces.
   (b) Find Lagrange equation of a rigid body in rotation about a stationary axis.
   (c) Prove that total energy of a conservative system does not change when the system is in motion.

4. (a) Define Poisson brackets. State and prove Jacobi-Poisson theorem.
   (b) Derive Routh's equation.
   (c) State and prove Hamilton principle.

5. (a) Derive Whittaker's equations.
   (b) State and prove Jacobi theorem.
   (c) Discuss the canonical character of a transformation in terms of Poisson Brackets.

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2018

Programme	: M.Sc. Mathematics (Re-Join) (Re-Appear)	Session	: 2015 – 16
Semester	: Third	Max. Time	: 3 Hours
Course Title	: Number Theory	Maximum Marks	: 60
Course Code	: SPMMAT 01 03 CC 04 3104		

Note: Attempt any five questions.

1. (a) Find the solution of the system of linear congruence [6]

$$3x + 7y \equiv 5 \ (mod\ 13)$$
$$2x + 5y \equiv 5 \ (mod\ 13).$$

   (b) Prove that primes are infinite. [6]

2. (a) When a Diophantine equation $ax + by = c$ has solutions? How many solutions exist? Find all solutions of $162x + 138y = 600$. [6]
   (b) State Fermat's and Wilson's Theorem. [4]
   (c) Define G.C.D and L.C.M of two numbers. [2]

3. (a) What is lifting of a solution? Check whether the solution $\alpha = 1$ of $x^3 + x + 3 \equiv 0\ (mod\ 5)$ can be lifted to some solution of $x^3 + x + 3 \equiv 0\ (mod\ 5^2)$. [6]
   (b) Show that for $n > 2$, Euler function $\phi(n)$ is even. [6]

4. (a) Define indices. Solve $x^3 \equiv 13(mod 15)$ using indices. [6]
   (b) State Mobius inversion formula. Show that Mobius function $\mu(n)$ is multiplicative. [3]
   (c) Find all primitive roots, if exist, of 12 and 13. [3]

5. (a) State Quadratic reciprocity law. Find all values of p so that $(3|p) = 1$. [6]
   (b) Define simple continued fraction of a real number. Which real number is represented by infinite continued fraction $[1; 1, 1, 1, \cdots]$. Find continued fraction of $\frac{22}{7}$. [6]

6. (a) Define Legendre symbool (a/p). Evaluate Legendre Symbol value for $(-28|33)$. [6]
   (b) State and prove Gauss Lemma. [6]

7. (a) Define convergents of a irrational number. Show that convergents are best rational approximation of the corresponding irrational number. [6]
   (b) Show that a real number is rational iff its simple continued fraction is of finite length. [6]

8. (a) Find the rational approximation to $e = [2; 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, \cdots]$ correct to four decimal places. [6]
   (b) Find $\frac{a}{b}$ with $0 < b \leq 5$ such that $|\sqrt{7} - \frac{a}{b}| \leq \frac{1}{6b}$. [6]

CENTRAL UNIVERSITY OF HARYANA
Term End Examinations, May/June-2018

Programme	: M.Sc. Mathematics		
Semester	: First **(Re-Appear)**	Session	: 2016-18
Course Title	: Abstract Algebra	Max. Time	: 3 Hours
Course Code	: SPMMAT 01 01 02 C 3104	Maximum Marks	: 70

Instructions:
1. Question no. 1 has seven sub parts and students need to answer any four. Each sub part carries three and half Marks.
2. Question no. 2 to 5 have three sub parts and students need to answer any two sub parts of each question. Each sub part carries seven marks.

1. (a) Show that $[Q(\sqrt{2}) : Q] = 2$. $(4 \times 3.5 = 14)$

   (b) Define Splitting field.

   (c) Define a solvable group.

   (d) Find the generators of a cyclic group of order 24.

   (e) Define algebraic extension.

   (f) Define algebraically closed field.

   (g) Define maximal subgroup.

2. (a) State and prove Cayley's theorem in group theory. [7]

   (b) State all the Sylow's theorem. Prove that a group of order 28 is not simple. [7]

   (c) If G is a non-abelian group of order p^3, where p is a prime number, show that $O[Z(G)] = p$. [7]

3. (a) State and prove Zassenhaus lemma. [7]

   (b) Prove that any two composition series of a finite group are equivalent. [7]

   (c) Prove that S_n is not solvable for $n \geq 5$. [7]

4. (a) Define minimal polynomial of an algebraic element. Show that $\sqrt{2} + \sqrt[3]{5}$ is algebraic over Q of degree 6. [7]

   (b) Let K be finite extension of field F, characteristic F is zero. Then K is a normal extension of F if and only if K is splitting field of some polynomial over F. [7]

   (c) If L is an algebraic extension of K and K is an algebraic extension of F, then prove that L is an algebraic extension of F. [7]

5. (a) Show that Galois group of a polynomial over a field is isomorphic to a subgroup of group of permutation of its root. [7]

   (b) Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals. [7]

   (c) If F is a field which contains all n^{th} root of unity for every positive integer n and if $p(x) \in F[x]$ is solvable by radicals over F, then the Galois group over F of $p(x)$ is a solvable group. [7]